

## Appendix W2.4.3

# Hydraulic Actuators

Hydraulic actuators obey the same fundamental relationships we saw in the water tank: continuity [Eq. (2.92)], force balance [Eq. (2.94)], and flow resistance [Eq. (2.95)]. Although the development here assumes the fluid to be perfectly incompressible, in fact, hydraulic fluid has some compressibility due primarily to entrained air. This feature causes hydraulic actuators to have some resonance because the compressibility of the fluid acts like a stiff spring. This resonance limits their speed of response.

### EXAMPLE W2.2

#### *Modeling a Hydraulic Actuator*

1. Find the nonlinear differential equations relating the movement  $\theta$  of the control surface to the input displacement  $x$  of the valve for the hydraulic actuator shown in Fig. W2.5.
2. Find the linear approximation to the equations of motion when  $\dot{y} = \text{constant}$ , with and without an applied load—that is, when  $F \neq 0$  and when  $F = 0$ . Assume  $\theta$  motion is small.

#### **Solution**

1. **Equations of motion:** When the valve is at  $x = 0$ , both passages are closed and no motion results. When  $x > 0$ , as shown in Fig. W2.5, the oil flows clockwise as shown and the piston is forced to the left. When  $x < 0$ , the fluid flows counterclockwise. The oil supply at high pressure  $p_s$  enters the *left* side of the large piston chamber, forcing the piston to the right. This causes the oil to flow out of the valve chamber from the rightmost channel instead of the left.

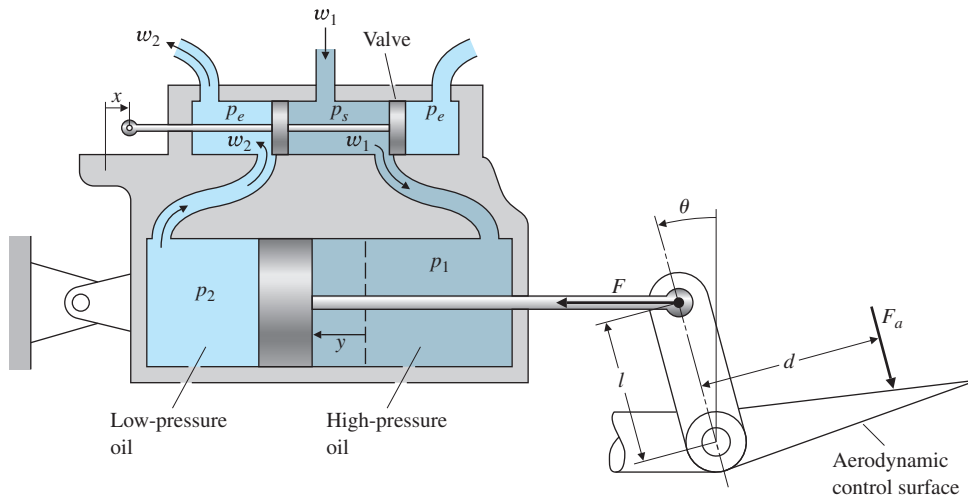


Figure W2.5

Hydraulic actuator with valve

We assume the flow through the orifice formed by the valve is proportional to  $x$ ; that is,

$$Q_1 = \frac{1}{\rho R_1} (p_s - p_1)^{1/2} x. \quad (2.101)$$

Similarly,

$$Q_2 = \frac{1}{\rho R_2} (p_2 - p_e)^{1/2} x. \quad (2.102)$$

The continuity relation yields

$$A \dot{y} = Q_1 = Q_2, \quad (2.103)$$

where

$$A = \text{piston area.}$$

The force balance on the piston yields

$$A(p_1 - p_2) - F = m \ddot{y}, \quad (2.104)$$

where

$m$  = mass of the piston and the attached rod,

$F$  = force applied by the piston rod to the control surface attachment point.

Furthermore, the moment balance of the control surface using Eq. (2.10) yields

$$I \ddot{\theta} = Fl \cos \theta - F_a d, \quad (2.105)$$

where

$I$  = moment of inertia of the control surface and attachment about the hinge,

$F_a$  = applied aerodynamic load.

To solve this set of five equations, we require the following additional kinematic relationship between  $\theta$  and  $y$ :

$$y = l \sin \theta. \quad (2.106)$$

The actuator is usually constructed so the valve exposes the two passages equally; therefore,  $R_1 = R_2$ , and we can infer from Eqs. (2.101) to (2.103) that

$$p_s - p_1 = p_2 - p_e. \quad (2.107)$$

These relations complete the nonlinear differential equations of motion; they are formidable and difficult to solve.

2. **Linearization and simplification:** For the case in which  $\dot{y}$  is a constant ( $\ddot{y} = 0$ ) and there is no applied load ( $F = 0$ ), Eqs. (2.104) and (2.107) indicate that

$$p_1 = p_2 = \frac{p_s + p_e}{2}. \quad (2.108)$$

Therefore, using Eq. (2.103) and letting  $\sin \theta = \theta$  (since  $\theta$  is assumed to be small), we get

$$\dot{\theta} = \frac{\sqrt{p_s - p_e}}{\sqrt{2}A\rho Rl}x. \quad (2.109)$$

This represents a single integration between the input  $x$  and the output  $\theta$ , where the proportionality constant is a function only of the supply pressure and the fixed parameters of the actuator. For the case  $\dot{y} = \text{constant}$  but  $F \neq 0$ , Eqs. (2.104) and (2.107) indicate that

$$p_1 = \frac{p_s + p_e + F/A}{2}$$

and

$$\dot{\theta} = \frac{\sqrt{p_s - p_e - F/A}}{\sqrt{2}A\rho Rl}x. \quad (2.110)$$

This result is also a single integration between the input  $x$  and the output  $\theta$ , but the proportionality constant now depends on the applied load  $F$ .

As long as the commanded values of  $x$  produce  $\theta$  motion that has a sufficiently small value of  $\ddot{\theta}$ , the approximation given by Eq. (2.109) or (2.110) is valid and no other linearized dynamic relationships are necessary. However, as soon as the commanded values of  $x$  produce accelerations in which the inertial forces ( $m\ddot{y}$  and the reaction to  $I\ddot{\theta}$ ) are a significant fraction of  $p_s - p_e$ , the approximations are no longer valid. We must then incorporate these forces into the equations, thus obtaining a dynamic relationship between  $x$  and  $\theta$  that is much more involved than the pure integration implied by Eq. (2.109) or (2.110). Typically, for initial control system designs, hydraulic actuators are assumed to obey the simple relationship of Eq. (2.109) or (2.110). When hydraulic

actuators are used in feedback control systems, resonances have been encountered that are not explained by using the approximation that the device is a simple integrator as in Eq. (2.109) or (2.110). The source of the resonance is the neglected accelerations discussed above along with the additional feature that the oil is slightly compressible due to small quantities of entrained air. This phenomenon is called the “oil-mass resonance.”

---

---

<sup>11</sup> Much of the background on Newton was taken from *Heisenberg Probably Slept Here*, by Richard P. Brennan, 1997. The book discusses his work and the other early scientists that laid the groundwork for Newton.